Mirror Symmetry of Landau–Ginzburg Orbifolds for Invertible Polynomials II

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Motivation

To a pair \((f, G)\) of a polynomial \(f\) and a certain finite group \(G\) of symmetries of \(f\), we want to associate an “orbifold version” of a Jacobian algebra in order to study an algebraic structure on the “first order deformation space of the pair \((f, G)\)”, the Hochschild cohomology group of the category of \(G\)-equivariant matrix factorizations of \(f\).
Preliminaries

Definition 1
The *Jacobian algebra* $\text{Jac}(f)$ of $f$ is a $\mathbb{C}$-algebra defined as

$$\text{Jac}(f) = \mathbb{C}[x_1, \ldots, x_N]/\left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_N}\right).$$

Set

$$\Omega_f := \Omega^N(\mathbb{C}^N)/(df \wedge \Omega^{N-1}(\mathbb{C}^N)).$$

By choosing a nowhere vanishing $N$-form $\zeta \in \Omega^N(\mathbb{C}^N)$ we have the following isomorphism

$$\vdash \zeta : \text{Jac}(f) \rightleftharpoons \Omega_f, \quad [\phi] \mapsto [\phi] \vdash \zeta := [\phi \zeta],$$

where $\zeta := [\widetilde{\zeta}] \in \Omega_f$. Namely, $\Omega_f$ is a $\text{Jac}(f)$-module of rank one.
Assume that \( \dim_{\mathbb{C}} \text{Jac}(f) < \infty \).

**Proposition 2 (The residue pairing)**

Define a \( \mathbb{C} \)-bilinear form \( J_f : \Omega_f \otimes_{\mathbb{C}} \Omega_f \rightarrow \mathbb{C} \) by

\[
J_f (\omega_1, \omega_2) := \text{Res}_{\mathbb{C}^N} \left[ \frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_N} \right] \cdot \left[ \phi \psi dx_1 \wedge \cdots \wedge dx_N \right],
\]

where \( \omega_1 = [\phi dx_1 \wedge \cdots \wedge dx_N] \) and \( \omega_2 = [\psi dx_1 \wedge \cdots \wedge dx_N] \).

Then, the bilinear form \( J_f \) on \( \Omega_f \) is non-degenerate.

In particular, \( J_f \) endows \( \text{Jac}(f) \) with a structure of a Frobenius algebra under \( \text{Jac}(f) \cong \Omega_f \) once we fix \( \tilde{\zeta} \).

**Remark 3**

An associative \( \mathbb{C} \)-algebra \((A, \circ)\) is called *Frobenius* if there is a non-degenerate \( \mathbb{C} \)-bilinear form \( \eta : A \otimes A \rightarrow \mathbb{C} \) such that

\[
\eta (X \circ Y, Z) = \eta (X, Y \circ Z) \text{ for } X, Y, Z \in A.
\]
Aim

To a pair \((f, G)\) of a polynomial \(f\) and a certain finite abelian group \(G\) of symmetries of \(f\), associate a \(\mathbb{Z}/2\mathbb{Z}\)-graded commutative Frobenius algebra \(\text{Jac}(f, G)\) which naturally generalize \(\text{Jac}(f)\).

Show the existence and uniqueness of the pair \((\text{Jac}(f, G), \Omega_{f,G})\) which generalize \((\text{Jac}(f), \Omega_f)\) with some expected properties.

Remark 4
Let \(MF(f)\) be the dg category of matrix factorizations of \(f\). Dyckerhoff shows

\[
HH^\bullet(MF(f)) \cong \text{Jac}(f), \quad HH_{\bullet+\mathbb{N}}(MF(f)) \cong \Omega_f.
\]

We are interested in the “calculus algebra” \((HH^\bullet, HH_\bullet)\) for \(G\)-equivariant (dg-)categories of matrix factorizations.
Invertible polynomials

Definition 5
A weighted homogeneous poly. $f \in \mathbb{C}[x_1, \ldots, x_N]$ is invertible if

1. $f$ is non-degenerate and the number of variables coincides with the number of monomials in the polynomial $f$, namely,

$$f(x_1, \ldots, x_N) = \sum_{i=1}^{N} c_i \prod_{j=1}^{N} x_j^{E_{ij}}, \quad c_i \in \mathbb{C}^*, \ E_{ij} \in \mathbb{Z}_{\geq 0}.$$

2. The matrix $E := (E_{ij})$ is invertible over $\mathbb{Q}$.

Remark 6
A weighted homogeneous polynomial $f$ is called non-degenerate if it has at most an isolated critical point at the origin in $\mathbb{C}^N$. One may assume that $c_i = 1$ for all $i$ by rescaling the variables.
Definition 7 (Berglund–Hübsch transposition)

For an invertible polynomial

\[ f(x_1, \ldots, x_N) = \sum_{i=1}^{N} c_i \prod_{j=1}^{N} x_j^{E_{ij}}, \quad c_i \in \mathbb{C}^*, \; E_{ij} \in \mathbb{Z}_{\geq 0}, \]

the polynomial

\[ \tilde{f} := \sum_{i=1}^{N} c_i \prod_{j=1}^{N} x_j^{E_{ji}}, \]

is called the \textit{Berglund–Hübsch transpose} of \( f \).
Proposition 8 (Kreuzer–Skarke)

An invertible polynomial $f$ can be written as a Thom–Sebastiani sum $f = f_1 \oplus \cdots \oplus f_p$ of invertible polynomials $f_\nu$, $\nu = 1, \ldots, p$ of the following types:

- $x_1^{a_1} x_2 + x_2^{a_2} x_3 + \cdots + x_{m-1}^{a_{m-1}} x_m + x_m^{a_m}$, chain type ($m \geq 1$),
- $x_1^{a_1} x_2 + x_2^{a_2} x_3 + \cdots + x_{m-1}^{a_{m-1}} x_m + x_m^{a_m} x_1$, loop type ($m \geq 2$).

A chain type with $m = 1$, $x_1^{a_1}$, is also called the Fermat type.

From now on, $f = f(x_1, \ldots, x_N)$ denotes an invertible polynomial.
Group of symmetries of $f$

Definition 9

The group of maximal diagonal symmetries of $f$ is defined as

$$G_f := \left\{ (\lambda_1, \ldots, \lambda_N) \in (\mathbb{C}^*)^N \mid f(\lambda_1 x_1, \ldots, \lambda_N x_N) = f(x_1, \ldots, x_N) \right\}$$

$$= \left\{ (\lambda_1, \ldots, \lambda_N) \in (\mathbb{C}^*)^N \mid \prod_{j=1}^{N} \lambda_j^{E_{1j}} = \cdots = \prod_{j=1}^{N} \lambda_j^{E_{Nj}} = 1 \right\},$$

Identify $G_f$ with the subgroup of diagonal matrices of $\text{GL}(N; \mathbb{C})$. 
**Definition 10 (Berglund–Henningson ’95)**

For a subgroup $G \subset G_f$, define

$$\tilde{G} := \text{Hom}(G_f / G, \mathbb{C}^*).$$

Berglund–Henningson expect that

$$(f, G) \overset{\text{mirror dual}}{\leftrightarrow} (\tilde{f}, \tilde{G}).$$

Now we know that the topological mirror symmetry holds.
Definition 11
Set

\[ \text{Fix}(g) := \{ x \in \mathbb{C}^N \mid g \cdot x = x \}, \quad N_g := \dim \mathbb{C} \text{Fix}(g), \]

\[ f^g := f|_{\text{Fix}(g)}. \]

Note that \( \text{Fix}(g) \) is a linear subspace of \( \mathbb{C}^N \).

Proposition 12
The polynomial \( f^g \) is also invertible. For each \( g \in G \), we have a natural surjective \( \mathbb{C} \)-algebra homomorphism \( \text{Jac}(f) \rightarrow \text{Jac}(f^g) \) and a natural \( \text{Jac}(f) \)-module structure on \( \Omega_{f^g} \).
Each element $g \in G$ has a unique expression of the form

$$g = \text{diag} \left( e \left[ \frac{a_1}{r} \right], \ldots, e \left[ \frac{a_N}{r} \right] \right)$$

with $0 \leq a_i < r$, where $r$ is the order of $g$ and

$$e[\ast] := \exp \left( 2\pi \sqrt{-1} \cdot \ast \right)$$

**Definition 13 (Ito–Reid)**

The *age* of $g \in G$ is the rational number defined by

$$\text{age}(g) := 1 \cdot \sum_{i=1}^{N} a_i.$$ 

If $G \subset G_f \cap \text{SL}(N; \mathbb{C})$, then $\text{age}(g)$ is an integer.
Definition of $\Omega_{f,G}$

Definition 14
Define a $\mathbb{Z}/2\mathbb{Z}$-graded $\mathbb{C}$-module $\Omega'_{f,G} = \left( \Omega'_{f,G} \right)_0 \oplus \left( \Omega'_{f,G} \right)_1$ by

$$(\Omega'_{f,G})_0 := \bigoplus_{g \in G} \Omega'_{f,g}, \quad (\Omega'_{f,G})_1 := \bigoplus_{g \in G} \Omega'_{f,g},$$

$\Omega'_{f,g} := \Omega_{fg}$.

Here, for $g \in G$ with $\text{Fix}(g) = \{0\}$ we define $\Omega_{fg} := \mathbb{C}1_g$ where $1_g$ is the constant function 1 on $\{0\}$.

The group $G$ acts on $\Omega_{fg}$ by the pull-back via its action on $\text{Fix}(g)$.

Definition 15

$$\Omega_{f,G} := \left( \Omega'_{f,G} \right)^G.$$
Definition 16 (The orbifold residue pairing)

Define a non-deg. \(\mathbb{Z}/2\mathbb{Z}\)-graded sym. \(\mathbb{C}\)-bilinear form on \(\Omega_{f,G}'\) by

\[
J_{f,G} := \bigoplus_{g \in G} J_{f,g}, \quad J_{f,g} : \Omega_{f,g}' \otimes \mathbb{C} \Omega_{f,g}^{-1} \to \mathbb{C},
\]

\[
J_{f,g} (\omega_g, \omega_{g^{-1}}) := e \left[ \frac{1}{2} \text{age}(g^{-1}) \right] |G| \begin{vmatrix}
\phi_g & \phi_{g^{-1}} dx_{i_1} \wedge \cdots \wedge dx_{i_{Ng}} \\
\frac{\partial f_g}{\partial x_{i_1}} & \cdots & \frac{\partial f_g}{\partial x_{i_{Ng}}}
\end{vmatrix},
\]

\[
J_{f,g} (1_g, 1_{g^{-1}}) := e \left[ \frac{1}{2} \text{age}(g^{-1}) \right] |G| \quad \text{if Fix}(g) = \{0\}.
\]

Here, \(\omega_\bullet = [\phi_\bullet dx_{i_1} \wedge \cdots \wedge dx_{i_{Ng}}] \in \Omega_{f,\bullet}', \bullet = g, g^{-1},\)
Definition 17 (The automorphism group of the pair \((f, G)\))
Define the group \(\text{Aut}(f, G)\) of automorphisms of \((f, G)\) as

\[
\text{Aut}(f, G) := \{ \varphi \in \text{Aut}_{\text{C-alg}}(\mathbb{C}[x]) \mid \varphi(f) = f, \varphi \circ g \circ \varphi^{-1} \in G \text{ for all } g \in G \}.
\]

Note that \(G\) is naturally identified with a subgroup of \(\text{Aut}(f, G)\).
The group \(\text{Aut}(f, G)\) acts naturally on \(\Omega'_{f, G}\) by

\[
\Omega'_{f, g} \longrightarrow \Omega'_{f, \varphi \circ g \circ \varphi^{-1}}, \quad \omega \mapsto \varphi^* \omega.
\]

Remark 18
We have \(\text{Aut}(f, G) = \text{Aut}_{\text{C-alg}}(\mathbb{C}[x] \ast G)\) where \(\mathbb{C}[x] \ast G\) is the skew group ring.
Definition 19

A *G-twisted Jacobian algebra* of \( f \) is a \( \mathbb{Z}/2\mathbb{Z} \)-graded \( \mathbb{C} \)-algebra \( \text{Jac}'(f, G) \) satisfying the following six axioms:

1. \( \exists \text{Jac}'(f, g) \cong \Omega'_{f,g} \), in particular, \( \text{Jac}'(f, \text{id}) = \text{Jac}(f) \) s.t.
   \[
   \text{Jac}'(f, G)_0 := \bigoplus_{g \in G, N-N_g \equiv 0 \text{ (mod 2)}} \text{Jac}'(f, g),
   \]
   \[
   \text{Jac}'(f, G)_1 := \bigoplus_{g \in G, N-N_g \equiv 1 \text{ (mod 2)}} \text{Jac}'(f, g).
   \]

2. The product \( \circ \) on \( \text{Jac}'(f, G) \) satisfies \( \text{Jac}'(f, g) \circ \text{Jac}'(f, h) \subseteq \text{Jac}'(f, gh) \) and the \( \mathbb{C} \)-subalgebra \( (\text{Jac}'(f, \text{id}), \circ) \) coincides with the \( \mathbb{C} \)-algebra \( \text{Jac}(f) \).
3. The \( \mathbb{C} \)-module \( \Omega'_{f,G} \) has a structure of a \( \text{Jac}'(f, G) \)-module

\[
\vdash: \text{Jac}'(f, G) \otimes \Omega'_{f,G} \longrightarrow \Omega'_{f,G}, \quad X \otimes \omega \mapsto X \vdash \omega,
\]

satisfying the following properties:

3.1 For any \( g, h \in G \) we have

\[
\text{Jac}'(f, g) \vdash \Omega'_{f,h} \subset \Omega'_{f,gh},
\]

and the \( \text{Jac}'(f, \text{id}) \)-module structure on \( \Omega'_{f,g} \) coincides with the \( \text{Jac}(f) \)-module structure on \( \Omega_{f,g} \).

3.2 By choosing a nowhere vanishing \( N \)-form \( \zeta \),

\[
\text{Jac}'(f, G) \xrightarrow{\approx} \Omega'_{f,G}, \quad \phi \mapsto \phi \vdash \zeta,
\]

where \( \zeta := [\tilde{\zeta}] \in \Omega'_{f,\text{id}} = \Omega_f \).

Namely, \( \Omega'_{f,G} \) is a free \( \text{Jac}'(f, G) \)-module of rank one.
4. The induced action of $\text{Aut}(f, G)$ on $\text{Jac}'(f, G)$ given by

$$\varphi^*(X) \mapsto \varphi^*(\zeta) := \varphi^*(X \mapsto \zeta), \quad \varphi \in \text{Aut}(f, G), \quad X \in \text{Jac}'(f, G).$$

We require that

$$\varphi^*(X) \circ \varphi^*(Y) = \varphi^*(X \circ Y), \quad \varphi \in \text{Aut}(f, G), \quad X, Y \in \text{Jac}'(f, G),$$

and it is $G$-twisted $\mathbb{Z}/2\mathbb{Z}$-graded commutative, namely, for $g, h \in G$ and $X \in \text{Jac}'(f, g)$, $Y \in \text{Jac}'(f, h)$,

$$X \circ Y = (-1)^{X \circ Y} g^*(Y) \circ X,$$

where $g^*$ is the induced action of $g \in G \subset \text{Aut}(f, G)$. 
5. For any \( g, h \in G \) and \( X \in \text{Jac}'(f, g) \), \( \omega \in \Omega'_{f,h} \), \( \omega' \in \Omega'_{f,G} \),

\[
J_{f,G}(X \vdash \omega, \omega') = (-1)^{\overline{X} \overline{\omega}} J_{f,G}(\omega, (h^{-1})^* X \vdash \omega'),
\]

where \( \overline{X} \) and \( \overline{\omega} \) are the \( \mathbb{Z}/2\mathbb{Z} \)-grading of \( X \) and \( \omega \).

6. Let \( G' \) be a subgroup of \( G_f \) such that \( G \subset G' \).

Fix a nowhere vanishing \( \mathcal{N} \)-form \( \zeta \).

By the axiom 3, the injective map \( \Omega'_{f,G} \rightarrow \Omega'_{f,G'} \) defined by

\[
\Omega'_{f,g} \rightarrow \Omega'_{f,g'}, \quad \omega \mapsto \omega,
\]

induces, an injective map of the \( \mathbb{Z}/2\mathbb{Z} \)-graded \( \mathbb{C} \)-modules \( \text{Jac}'(f, G) \rightarrow \text{Jac}'(f, G') \), which is an algebra-homomorphism.
There exists a $G$-twisted Jacobian algebra $\text{Jac}'(f, G)$ of $f$ unique up to isomorphism.

Lemma 21 (Key Lemma)
The group $G_f \cap \text{SL}(N; \mathbb{C})$ is a proper subgroup of $G_f$.

Definition 22
The $\mathbb{Z}/2\mathbb{Z}$-graded commutative Frobenius algebra

$$\text{Jac}(f, G) := (\text{Jac}'(f, G))^G$$

is called the \textit{orbifold Jacobian algebra} of $(f, G)$. 
Historical remarks

Certain works towards the definition of orbifold Frobenius algebras were also done previously by R. Kaufmann and M. Krawitz.

Kaufmann '03 proposed a notion of the orbifold Frobenius superalgebras ($\mathbb{Z}/2\mathbb{Z}$-algebras) for $(f, G)$. There one needs to fix a non-unique “choice of a two cocycle”.

Krawitz '09 gave a particular construction of an algebra, not a $\mathbb{Z}/2\mathbb{Z}$-graded algebra, for $(f, G)$. However, his “formula” could only be well-defined for weighted homogeneous polynomials. He did not consider the uniqueness of the algebra.

Our definition is valid for all $f$ with finite dimensional $\text{Jac}(f)$. Our axiom 4 on ”$\text{Aut}(f, G)$-invariance” chooses a particular two cocycle, which reproduces Krawitz’s “formula” if $\text{Jac}(f, G) = \text{Jac}(f, G)_0$ (no odd subspaces).
Product Formula

Let $I_g := \{i_1, \ldots, i_{N_g}\}$ be a subset of $\{1, \ldots, N\}$ such that $\text{Fix}(g) = \{x \in \mathbb{C}^N \mid x_j = 0, j \notin I_g\}$. In particular, $I_{id} = \{1, \ldots, N\}$. Denote by $I_g^c$ the complement of $I_g$ in $I_{id}$. For each $g \in G_f$, there exists $v_g \in \text{Jac}'(f, g)$ such that $
abla'(f, id)v_g = \nabla'(f, g)$ and for $g, h \in G_f$ with $I_g^c \cap I_h^c = \emptyset$

$$v_g \circ v_h = \tilde{e}_{g,h}v_{gh}, \quad \tilde{e}_{g,h} \in \{\pm 1\},$$

$$v_g \circ v_{g^{-1}} = (-1)^{\frac{1}{2}(N-N_g)(N-N_g-1)} \cdot e\left[-\frac{1}{2} \text{age}(g)\right] \cdot [H_{g,g^{-1}}]v_{id},$$

where

$$H_{g,g^{-1}} := c_g \cdot \det\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)_{i,j \in I_g^c}, \quad c_g \in \mathbb{C}^*.$$
Suppose that $f = f(x_1, x_2, x_3)$ defines an ADE singularity. The holomorphic map $f : \mathbb{C}^3 \rightarrow \mathbb{C}$ yields $\tilde{f} : \mathbb{C}^3/G \rightarrow \mathbb{C}$. We can find a chart $U(\cong \mathbb{C}^3) \subset \mathbb{C}^3/G$ containing all the critical points of $\tilde{f}$. Set $\bar{f} := \tilde{f}|_U$.

**Theorem 23 (BTW)**

*We have an isomorphism of Frobenius algebras*

$$\text{Jac}(f, G) \cong \text{Jac}(\bar{f}).$$

**Remark 24**

There is an equivalence

$$HMF^G(f) \cong HMF(\bar{f}).$$
An example of an orbifold Jacobian algebra

\[ f := x_1^3 + x_2^3 + x_3^2, \quad G := \langle g \rangle, \quad g := \left( \exp \left( \frac{2\pi \sqrt{-1}}{3} \right), \exp \left( \frac{4\pi \sqrt{-1}}{3} \right), 1 \right). \]

\[ \text{Jac}(f) \cong \mathbb{C}[x_1, x_2, x_3]/(x_1^2, x_2^2, x_3) \cong \langle [1], [x_1], [x_2], [x_1 x_2] \rangle_{\mathbb{C}}. \]

\[ J_f([dx_1 \wedge dx_2 \wedge dx_3], [x_1 x_2 dx_1 \wedge dx_2 \wedge dx_3]) = \frac{1}{18}. \]

We have

\[ \text{Jac}(f, G) \cong \langle [1], [x_1 x_2] \rangle_{\mathbb{C}} \oplus \langle e_g, e_{g^{-1}} \rangle_{\mathbb{C}}, \]

where \( e_g \in \text{Jac}(f, g) \) is mapped to \([dx_3] \in \Omega_{f,g} = \Omega_{fg} \).

\[ J_{f, \text{id}}([dx_1 \wedge dx_2 \wedge dx_3], [x_1 x_2 dx_1 \wedge dx_2 \wedge dx_3]) = 3 \cdot \frac{1}{18} = \frac{1}{6}, \]

\[ J_{f, g}([dx_3], [dx_3]) = -1 \cdot \frac{1}{3} \cdot \frac{1}{2} = -\frac{1}{6}. \]

\[ \implies e_g \circ e_{g^{-1}} = -[x_1 x_2], \quad e_g^2 = 0, \quad e_{g^{-1}}^2 = 0. \]
On the other hand, we have $\bar{f} = y_1^2 + y_2^2 y_3 + y_3^2 y_2$.

\[
\text{Jac}(\bar{f}, \{\text{id}\}) = \text{Jac}(\bar{f}) = \mathbb{C}[y_1, y_2, y_3]/(y_1, 2y_2 y_3 + y_3^2, y_2^2 + 2y_2 y_3)
\]
\[
\cong \mathbb{C}[y_2, y_3]/(2y_2 y_3 + y_3^2, y_2^2 + 2y_2 y_3).
\]

\[
J_{\bar{f},\{\text{id}\}}([dy_1 \wedge dy_2 \wedge dy_3], [y_2 y_3 dy_1 \wedge dy_2 \wedge dy_3]) = \frac{1}{6}.
\]

Therefore, by the following correspondence

\[
[y_2] \mapsto e^{\frac{2\pi \sqrt{-1}}{3}} e_g + e^{\frac{4\pi \sqrt{-1}}{3}} e_{g-1}, \quad [y_3] \mapsto e^{\frac{4\pi \sqrt{-1}}{3}} e_g + e^{\frac{2\pi \sqrt{-1}}{3}} e_{g-1},
\]

we have an isomorphism of Frobenius algebras

\[
\text{Jac}(\bar{f}, \{\text{id}\}) \cong \text{Jac}(f, G).
\]

Let $f_1, f_2 \in \mathbb{C}[x, y, z]$ be invertible polynomials defining Arnold’s 14 exceptional singularities. There exists an isomorphism of Frobenius algebras

$$\text{Jac}(f_1) \cong \text{Jac}(\tilde{f}_2, G_{\tilde{f}_2}^{\text{SL}}), \quad G_{\tilde{f}_2}^{\text{SL}} := G_{\tilde{f}_2} \cap \text{SL}(3; \mathbb{C})$$

if and only if the associated singularities of $f_1$ and $f_2$ are strange dual to each other in the sense of Arnold.

Remark 26

The equivalence

$$\text{HMF}(f_1) \cong \text{HMF}^{G_{\tilde{f}_2}^{\text{SL}}}(\tilde{f}_2)$$

is given by Carqueville-Ros Camacho-Runkel and Newton-Ros Camacho.
$G$-equivariant Matrix Factorizations

Theorem 27 (Basalaev–T–Werner, in preparation)

We have the $\mathbb{C}$-algebra isomorphism

$$HH^\bullet(\text{MF}^G_{\mathbb{C}[x]}(f)) \cong \text{Jac}(f, G).$$

In the proof, we use:

- Shklyarov’s algebra (arXiv:1708.06030) whose $G$-invariant part is $HH^\bullet(\text{MF}^G_{\mathbb{C}[x]}(f))$.

- Reduction to the isomorphism for $(f, G_f)$ where $f$ is of chain type or of loop type by the isomorphism

  $$\text{Jac}'(f_1 + f_2, G_{f_1+f_2}) \cong \text{Jac}'(f_1, G_{f_1}) \otimes \text{Jac}'(f_2, G_{f_2}).$$

- Direct calculation of the product formula of Shklyarov’s algebra.
Thank you very much!