Gravity from Entanglement and RG Flow in a Top-down Approach

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In collaboration with

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Outline

- gauge/gravity duality in a top-down approach
- Holographic entanglement entropy
- Construction of 4-dimensional LLM
- Holography and entanglement entropy
- Einstein equation from entanglement entropy of nonconformal field theory
- Summary

AdS/CFT correspondence by Maldacena (1997)

QFT with fixed spacetime

Quantum gravity

Exactly equivalent

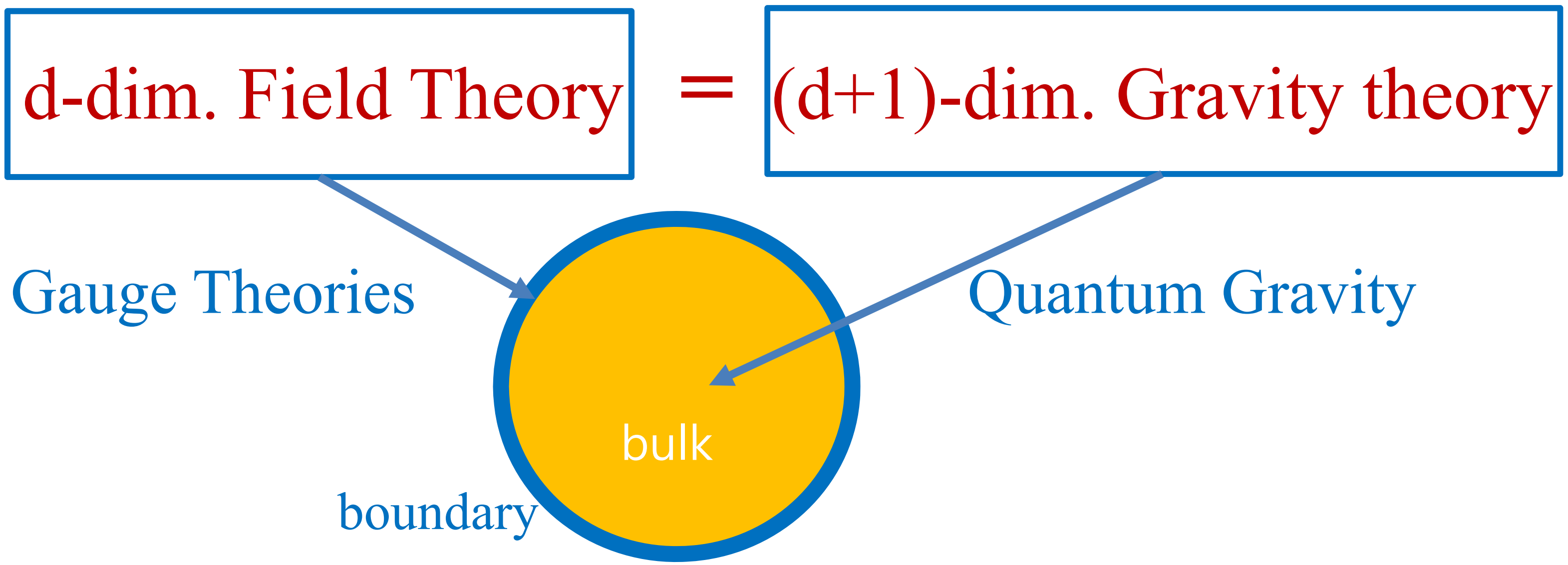
$$Z[QFT] = Z[Gravity]$$

GKP-W relation (1998)

Examples are very rare!!

Gauge/gravity

- Gauge/gravity duality



- After the conjecture by Maldacena in 1997, there were many works in this direction, such as string theory, QCD, nulcear physics, condensed matter physics, cosmology, etc.

Gauge/gravity

- Duality properties of field theory and gravity theory

Quantum field theory ⟺ (duality) ⟺ Quantum gravity

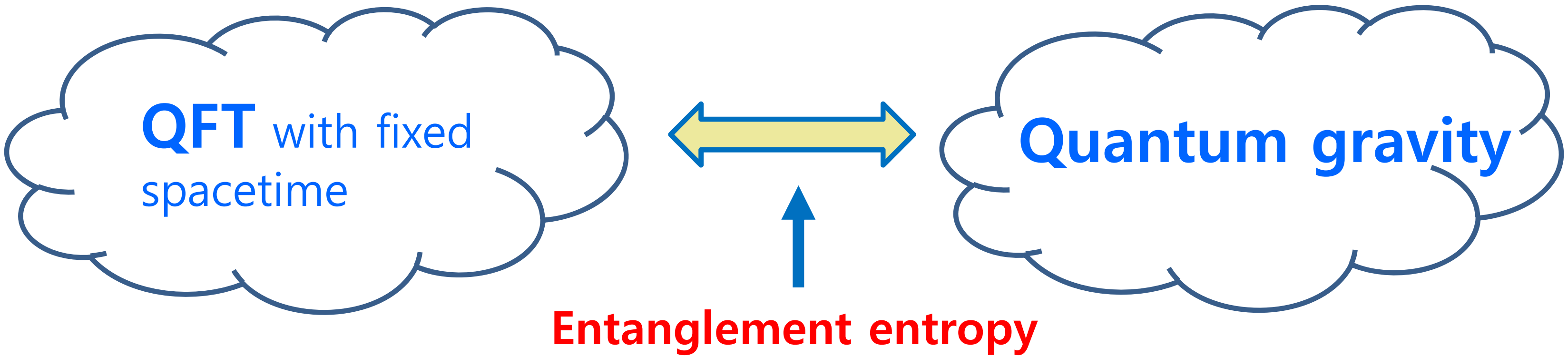
Strongly coupled gauge theory ⟺ (Large N) ⟺ Weakly curved gravity (classical gravity)

- **Very useful but difficult to check the duality!**
  For some BPS objects which have no quantum corrections, it is possible to check the duality using supersymmetry and conformal symmetry **in the large N limit.**

What is origin of the gauge/gravity duality?
Any proof for the duality?

→ No concrete answer for these questions though there are many evidences!!

One clue from entanglement entropy!



Holographic EE
Emergent gravity from EE
cMERA
Quantum error correction
....

Entanglement Entropy

- **Quantum entanglement** is a physical phenomenon that occurs when pairs of particles interact. Then the quantum state of each particle cannot be described independently.
  ➔ **entanglement entropy (EE)**

- **Density matrix of a ground state** $|\Psi\rangle$ **:**

$$\rho_{tot} = |\Psi\rangle\langle\Psi|$$

- **Reduced density matrix:**

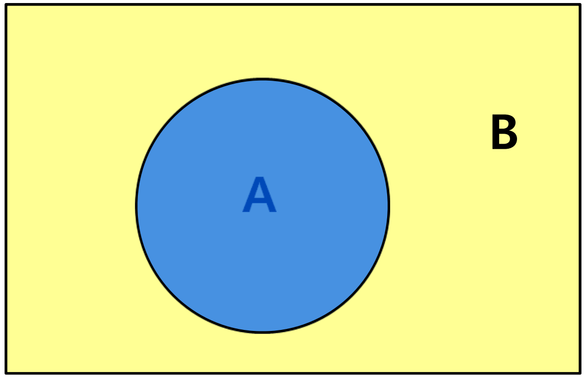
$$\rho_A = \mathrm{Tr}_B[\rho_{tot}]$$

- **Entanglement entropy (EE) = von Neumann entropy**

$$S_A = -\mathrm{Tr}_A\, \rho_A \ln \rho_A$$

Entanglement Entropy

- In QFT?



$$\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$$

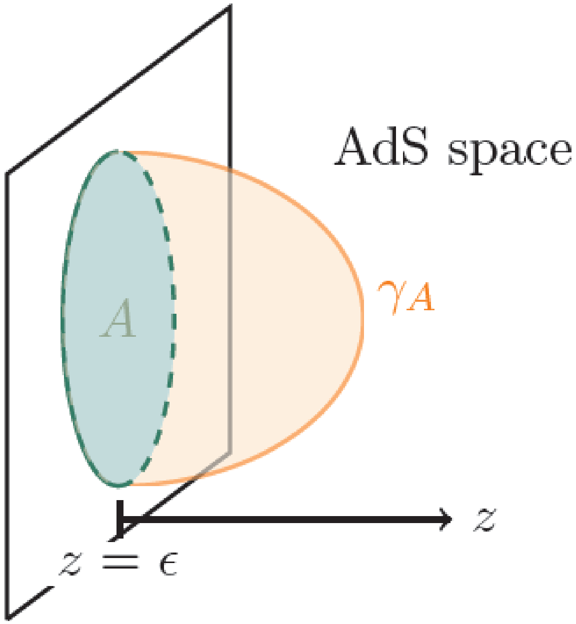
- Replica trick for the Renyi entropy in path integral method

Holographic entanglement entropy (HEE)

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$

$$ds^2_{\mathrm{AdS}} = R^2_{\mathrm{AdS}} \frac{-dt^2 + dx^i dx^i + du^2}{u^2}$$

[Ryu-Takayanagi 06]

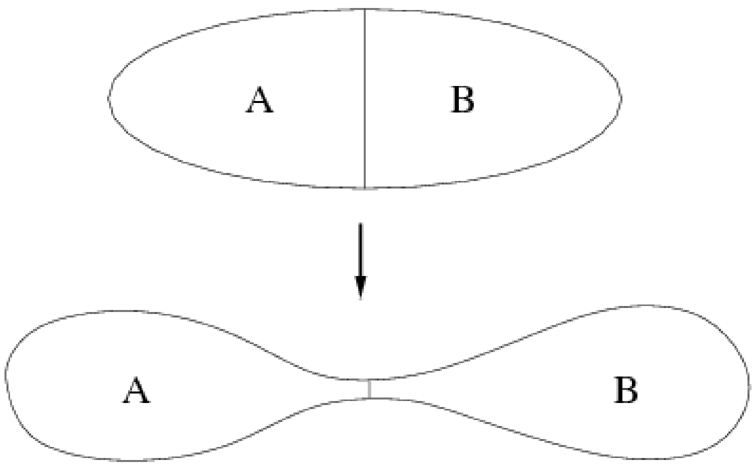


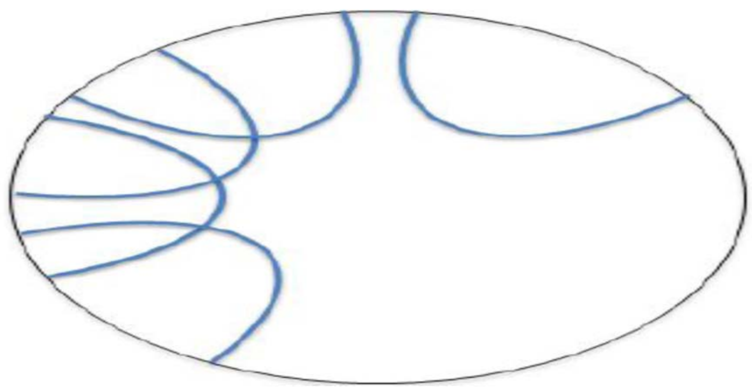
γ_A is the codimension 2 minimal surface

This HEE proposal can be applied to any dual geometry which has boundary!!

Emergence of gravity from gauge theory

- One clue can come from entanglement entropy!





- Without entanglement, the bulk region may disappear.

Entanglement entropy 1st law

- Small fluctuation around the vacuum

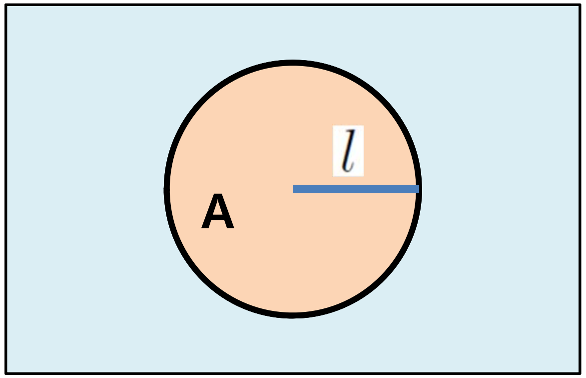
$$|\psi\rangle \longrightarrow |\psi\rangle + \delta|\psi\rangle$$

$$\begin{aligned} \delta S_A &= -\mathrm{tr}\left(\delta\rho_A \log \rho_A\right) \\ &= \mathrm{tr}\left(\delta\rho_A H_A\right) \\ &= \delta\langle H_A\rangle = \delta E_A \end{aligned}$$

$$\rho_A = e^{-H_A}$$

Entanglement entropy 1st law

- Ball-shaped region A



$\rho_A = e^{-H_A}$ **modular Hamiltonian**

$$H_A = 2\pi \int d^d x \frac{l^2 - r^2}{2l} T_{00}(x)$$

$$\delta S_A = 2\pi \int d^d x \frac{l^2 - r^2}{2l} \delta \langle T_{00} \rangle \equiv \delta E_{hyp}$$

hyperbolic energy

Entanglement entropy 1st law

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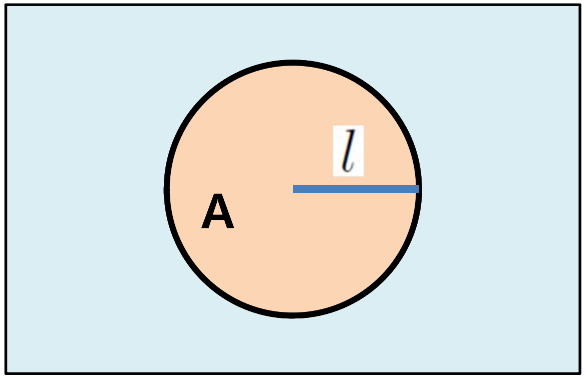

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hyperbolic energy

Gauge/gravity duality
RT formula ⇕ [Raamsdonk et al, 2013]

Linearized Einstein equation

How about non-conformal CFT?

Corresponding 1st law for EE?
Extended 1st law?
Emergent gravity?

$$\delta G_{\mu\nu} = 8\pi G_N T_{\mu\nu} ??$$

Mass-deformed ABJM and LLM

ABJM (CFT) + mass deformation = mABJM (non-CFT)

Vacua of mABJM ⟺ LLM geometry with $\mathbb{Z}_k$ orbifold

[Lin-Lunin-Maldacena, 2004]

$$\mathcal{M}_1^{(n)} = \begin{pmatrix} \sqrt{n} & 0 & & & & \\ & \sqrt{n-1} & 0 & & & \\ & & \ddots & \ddots & & \\ & & & \sqrt{2} & 0 & \\ & & & & 1 & 0 \end{pmatrix}$$

$$\mathcal{M}_2^{(n)} = \begin{pmatrix} 0 & 1 & & & & \\ & 0 & \sqrt{2} & & & \\ & & \ddots & \ddots & & \\ & & & 0 & \sqrt{n-1} & \\ & & & & 0 & \sqrt{n} \end{pmatrix}$$

Discrete Higgs vacua denoted by numerical values

➔Quantitative comparison is possible!!

[Gomis et al 08]

[Cheon-Kim-Kim 2011]

Gauge/gravity duality and entanglement entropy (EE)

- **Relevant deformation** by inserting relevant operators to the CFT (i.e. mass deformation) ➔ non-conformal deformation

ABJM(3d CFT) **+ relevant deformations**

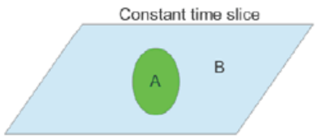
correspondence

asymptotically $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ Solution in 11d SUGRA

(GKP-W relation) Gauge/gravity mapping for $\langle \mathcal{O}(x) \rangle$ and $\langle \mathcal{O}(x)\mathcal{O}(y)\ldots \rangle$

KK reduction

asymptotically AdS_4 solutions in 4d gravity of KK modes



EE for an entangling surface (replica method)

RT conjecture in the large N limit

HEE deformed from the AdS case

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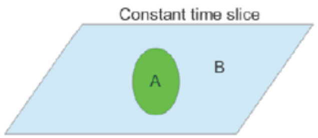

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HEE deformed from the AdS case

Construction of 4-dimensional gravity

EOM for graviton mode in 11d: (up to μ_0^2-order)

$$\left(L_E + \frac{12}{L^2}\right)\hat{h}^0_{\mu\nu} + \frac{1}{34560}\Big\{ -\frac{26}{3}\nabla_\mu\check{\psi}^2\nabla_\nu\check{\psi}^2 + \frac{28}{3}\check{\psi}^2\nabla_\mu\nabla_\nu\check{\psi}^2 + \frac{L^2}{3}\nabla_\mu\nabla^\rho\check{\psi}^2\nabla_\nu\nabla_\rho\check{\psi}^2$$
$$+ \frac{L^2}{2}\nabla^\rho\check{\psi}^2\nabla_\mu\nabla_\nu\nabla_\rho\check{\psi}^2 + \frac{L^4}{24}\nabla_\mu\nabla^\rho\nabla^\sigma\check{\psi}^2\nabla_\nu\nabla_\rho\nabla_\sigma\check{\psi}^2 + \frac{L^4}{32}\nabla^\rho\nabla^\sigma\check{\psi}^2\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma\check{\psi}^2$$
$$- g_{\mu\nu}\left(\frac{12}{L^2}\check{\psi}^2\check{\psi}^2 + \nabla^\rho\check{\psi}^2\nabla_\rho\check{\psi}^2 + \frac{35L^2}{48}\nabla^\rho\nabla^\sigma\check{\psi}^2\nabla_\rho\nabla_\sigma\check{\psi}^2 - \frac{L^4}{64}\nabla^\tau\nabla^\rho\nabla^\sigma\check{\psi}^2\nabla_\tau\nabla_\rho\nabla_\sigma\check{\psi}^2\right)\Big\}$$
$$+ \frac{L^2}{48}\left(\nabla_\mu\nabla_\nu t^1_+ t^1_+ + \frac{1}{2}\nabla_\mu t^1_+\nabla_\nu t^1_+\right) + \frac{L^2}{96}g_{\mu\nu}\left(\nabla_\rho t^1_+\nabla^\rho t^1_+ - \frac{16}{L^2}t^1_+ t^1_+\right) = 0.$$

Linear order

quadratic order and higher derivative terms appears

$$\hat{h}^0_{\mu\nu} \equiv h^0_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\phi^0 + \frac{1}{24}g_{\mu\nu}\hat{\psi}^0.$$

$$L_E h^0_{\mu\nu} = \tfrac{1}{2}\left(-\Box h^0_{\mu\nu} + \nabla^\rho\nabla_\mu h^0_{\nu\rho} + \nabla^\rho\nabla_\nu h^0_{\mu\rho} - \nabla_\mu\nabla_\nu h^0\right)$$

EOM for scalar modes in 11d:

$$\left(\Box - M_t^2\right)t^1_+ = 0, \qquad \left(\Box - M_\psi^2\right)\check{\psi}^2 = 0. \qquad M_t^2 = M_\psi^2 = -\tfrac{8}{L^2}$$

Construction of 4-dimensional gravity

EOM for graviton mode in 4d: (up to μ_0^2-order)

$$\left(L_E + \frac{12}{L^2}\right)H_{\mu\nu} + 8\pi G_N A_t\left(\nabla_\mu T\nabla_\nu T + \frac{M_t^2}{2}g_{\mu\nu}T^2\right) + 8\pi G_N A_\psi\left(\nabla_\mu\Psi\nabla_\nu\Psi + \frac{M_\psi^2}{2}g_{\mu\nu}\Psi^2\right) = 0.$$

$$L_E H_{\mu\nu} = \frac{1}{2}\left(-\Box H_{\mu\nu} + \nabla^\rho\nabla_\mu H_{\rho\nu} + \nabla^\rho\nabla_\nu H_{\rho\mu} - \nabla_\mu\nabla_\nu H\right)$$

- **To absorb higher derivatives, we needs field redefinition:**

$$H_{\mu\nu} = \hat{h}^0_{\mu\nu} + g_{\mu\nu}\left(C_1\check{\psi}^2\check{\psi}^2 + C_2\nabla^\rho\check{\psi}^2\nabla_\rho\check{\psi}^2\right) + C_3\nabla_\mu\check{\psi}^2\nabla_\nu\check{\psi}^2 + g_{\mu\nu}C_4\nabla^\rho\nabla^\sigma\check{\psi}^2\nabla_\rho\nabla_\sigma\check{\psi}^2 + C_5\nabla_\mu\nabla^\rho\check{\psi}^2\nabla_\nu\nabla_\rho\check{\psi}^2 + g_{\mu\nu}C_t t^1_+ t^1_+.$$

$$C_1 = -\frac{1}{40}\frac{1}{2^3\times 3^3},\quad C_2 = -\frac{1}{40}\frac{L^2}{2^8\times 3^3},\quad C_3 = -\frac{1}{40}\frac{7L^2}{2^8\times 3^4},\quad C_4 = -\frac{1}{40}\frac{L^4}{2^{11}\times 3^3},$$

$$C_5 = -\frac{1}{40}\frac{L^4}{2^{10}\times 3^4},\quad C_t = -\frac{L^2}{2^5\times 3},\quad 8\pi G_N A_t = -\frac{L^2}{2^5\times 3},\quad 8\pi G_N A_\psi = -\frac{1}{2^8\times 3^2}.$$

Construction of 4-dimensional gravity

- Near the UV fixed point where the ABJM theory lives on, (in the small mass expansion,) the dual 4-dimensional gravity action is given by

$$S = \frac{1}{16\pi G_N^{(4)}} \int d^4x \sqrt{-g} \left(\hat{R} - 2\Lambda \right) + S_m$$

$$S_m = -\frac{A_t}{2} \int d^4x \sqrt{-g} (\nabla_\mu T \nabla^\mu T + M_t^2 T^2) - \frac{A_\psi}{2} \int d^4x \sqrt{-g} (\nabla_\mu \Psi \nabla^\mu \Psi + M_\psi^2 \Psi^2)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$$\left(\Box - M_T^2\right) T = 0$$

$$\left(\Box - M_\psi^2\right) \psi = 0$$

Asymptotically AdS solution with relevant deformations

- Fluctuations around AdS_4 space $g_{\mu\nu} = g^{(0)}_{\mu\nu} + H_{\mu\nu}$

Exact holography

- Dictionary of the gauge/gravity duality (GKP-W relation)

$$\langle \mathcal{O}^{(\Delta)} \rangle = \mathbb{N}\phi_{(\Delta)}.$$ $\mathbb{N}$ is a normalization factor.

$$\langle \mathcal{O}^{(1)} \rangle = -24\mathbb{N}\mu_0\beta_3 \qquad \mathcal{O}^{(1)} = \frac{1}{2\sqrt{2}}\mathrm{Tr}\,(Z^a Z_a^\dagger - W^{\dagger a} W_a)$$

- **Vevs of the CPO with conformal dimension 1 in mABJM**

$$\langle \mathcal{O}^{(\Delta=1)} \rangle = \frac{\mu k}{4\sqrt{2}\pi}\sum_{n=0}^{\infty}(N_n - N_n')n(n+1)$$

➔ Field theory calculation in the large N limit

$$\mathbb{N} = -\frac{N^{\frac{3}{2}}}{72\sqrt{2}\pi}$$

$$\{l_n, l_n'\} \Longleftrightarrow \{N_n, N_n'\}$$

$$\sum_{n=0}^{\infty}\left[n(n+1)(l_n - l_n')\right] = \frac{N^{3/2}}{3}\beta_3,$$

$$\langle \mathcal{O}^{(1)} \rangle = \frac{N^{\frac{3}{2}}\mu_0}{3\sqrt{2}\,\pi}\beta_3$$

➔ GKP-W relation

➔ Exact holography for all droplets

Gauge/gravity duality and entanglement entropy (EE)

- **Relevant deformation** by inserting relevant operators to the CFT (i.e. mass deformation) ➔ non-conformal deformation

ABJM(3d CFT)
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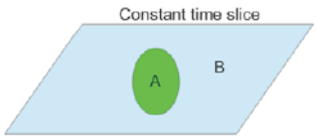

EE for an entangling surface (replica method)

RT conjecture
in the large N limit

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Holographic entanglement entropy

- Metric fluctuations around AdS_4 and induced metric:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + H_{\mu\nu} \qquad \tilde{g}_{ij} = \frac{\partial x^\mu}{\partial \sigma^i}\frac{\partial x^\nu}{\partial \sigma^j} g_{\mu\nu} = \tilde{g}^{(0)}_{ij} + \tilde{H}_{ij}$$

$$S = \frac{1}{4G_N}\int d^2\sigma \sqrt{\det \tilde{g}_{ij}} \qquad \delta S = \frac{1}{8G_N}\int d^2\sigma \sqrt{\det \tilde{g}^{(0)}_{ij}}\; \tilde{g}^{(0)ij}\tilde{H}_{ij}$$

$$H_{ij} = -\frac{L^2\mu_0^2}{180}\left(30+\beta_3^2\right)\eta_{ij} + \mathcal{O}\left(\mu_0^3\right), \quad (i,j=0,1,2) \qquad H_{zz} = -\frac{L^2\mu_0^2}{1440}\left(960+29\beta_3^2\right) + \mathcal{O}\left(\mu_0^3\right)$$

$$\psi = -24\beta_3\mu_0 z + \mathcal{O}\left(\mu_0^2\right), \qquad T = 16\sqrt{3}\mu_0 z + \mathcal{O}\left(\mu_0^2\right)$$

Holographic entanglement entropy

- Variation of the EE is determined by **source and vev**

$$\delta S = -\frac{\pi L^2(\mu_0 l)^2}{8G_N}\left(\frac{4}{3}+\frac{1}{24}\beta_3^2\right) = -\frac{4\sqrt{2}\pi\, N^2 l^2}{\sqrt{\lambda}}\left[\frac{1}{9}\left(J_{\tilde{\mathcal{O}}^{(2)}}\right)^2+\frac{1}{16}\left(\frac{\pi\sqrt{\lambda}\langle\mathcal{O}^{(1)}\rangle_m}{N^2}\right)^2\right]$$

This result exactly reproduces the HEE using PDE from the 11d LLM. [Kim-Kim-OK 2016]

$$\langle\mathcal{O}^{(1)}\rangle = \frac{N^{\frac{3}{2}}\mu_0}{3\sqrt{2}\,\pi}\,\beta_3$$

$$J_{\tilde{\mathcal{O}}^{(2)}} = \mu_0$$

How can we interpret this result?

Variation of entanglement entropy

- EE calculation **using path integral method** in CFT with relevant perturbation

$$I = I^{(0)} + \lambda \int d^d w\, \mathcal{O}^{(\Delta)}$$

$$\delta S_A = \langle H_A \rangle_{\tilde{\lambda}} - 2\pi \int d\Sigma^\mu \xi^\nu \tilde{T}_{\mu\nu} + S_{ct}.$$

[2014 Faulkner]

$$\tilde{T}_{\mu\nu} = \nabla_\mu \tilde{\phi} \nabla_\nu \tilde{\phi} - \frac{1}{2} g^{(0)}_{\mu\nu} \left(\nabla_\lambda \phi \nabla^\lambda \tilde{\phi} + m^2 \tilde{\phi}^2 \right)$$

$$\nabla^2 \tilde{\phi} - \frac{\Delta(\Delta - d)}{L^2_{\mathrm{AdS}}} \tilde{\phi} = 0$$

Variation of entanglement entropy

- In the mass-deformed ABJM theory, there are two relevant operators which are dual to Ψ and T

$$\delta S_A = -2\pi \int d\Sigma^\mu \xi^\nu \tilde{T}_{\mu\nu}$$

$$= \delta S_\Psi^{(2)} + \delta S_T^{(2)} = -128\pi^2 L^2 \left(A_t + \frac{3A_\psi \beta_3^2}{4} \right) (\mu_0 l)^2$$

$$\tilde{\Psi} = -24\sqrt{A_\psi}\,\beta_3 \mu_0 z + \mathcal{O}(\mu_0^3), \qquad \tilde{T} = 16\sqrt{3}\sqrt{A_t}\,\mu_0 z + \mathcal{O}(\mu_0^3)$$

↑

Gauge/gravity duality

Variation of entanglement entropy

$$\delta\gamma_A = -\int d\Sigma^\mu \xi^\nu \delta G_{\mu\nu} + \delta\gamma_A^{(ct)}$$

[Iyer and Wald 1994]

$$\delta G_{\mu\nu} = \frac{1}{2}\Big(- \Box H_{\mu\nu} + \nabla^\rho\nabla_\mu H_{\rho\nu} + \nabla^\rho\nabla_\nu H_{\rho\mu} - \nabla_\mu\nabla_\nu H\Big) + \frac{12}{L^2}H_{\mu\nu} - \frac{6}{L^2}g_{\mu\nu}H$$
$$-\frac{1}{2}g_{\mu\nu}(\nabla^\rho\nabla^\sigma H_{\rho\sigma} - \Box H).$$

$$H_{ij} = -\frac{L^2\mu_0^2}{180}\left(30+\beta_3^2\right)\eta_{ij} + \mathcal{O}\left(\mu_0^3\right), \quad (i,j=0,1,2) \qquad H_{zz} = -\frac{L^2\mu_0^2}{1440}\left(960+29\beta_3^2\right) + \mathcal{O}\left(\mu_0^3\right)$$

$$\psi = -24\beta_3\mu_0 z + \mathcal{O}\left(\mu_0^2\right), \qquad T = 16\sqrt{3}\mu_0 z + \mathcal{O}\left(\mu_0^2\right)$$

$$-\int_{\mathcal{H}_0} d\Sigma^t\xi^t\delta G_{tt} = -\frac{\pi L^2}{48}\left(32+\beta_3^2\right)(\mu_0 l)^2 + \frac{\pi L^2}{128}\frac{l}{z_\Lambda}\left(32+\beta_3^2\right)(\mu_0 l)^2$$

Variation of entanglement entropy

$$\delta S_A = -2\pi \int d\Sigma^\mu \xi^\nu \tilde{T}_{\mu\nu} = -\frac{1}{4G_N^{(4)}} \int d\Sigma^\mu \xi^\nu \delta G_{\mu\nu} = \frac{\delta A}{4G_N}$$

Gauge/gravity duality
RT formula

$$\delta G_{\mu\nu} = 8\pi G_N^{(4)} \tilde{T}_{\mu\nu}.$$

Einstein equation from EE of non-conformal field theory

Conclusion

- Using the LLM geometry in 4-dim. gravity:
  1. $\delta S = \delta S(\text{source, vev})$
  2. extended thermodynamic-like first law of $\delta S = \delta E$ for all droplet solution in non-conformal field theory
  3. $\delta S \leftrightarrow g_{\mu\nu}$ relation using exact holography and the entropic counterpart of the Einstein equation

- Next order for the mass parameter
- IR entanglement entropy for the mABJM and LLM

Thank you!!