From Electron Gas To Wigner Solid

Consider a system of electrons with spin-polarization with a uniform positive background charges, i.e., a jellium model.

(a) Find the ground-state energy to 1st order as a function of the spin polarization $\zeta = (N_+ - N_-)/N$ and the density $n = N/V$.

(b) Show that the ferromagnetic state ($\zeta = 1$) becomes a lower energy state than the non-magnetic state ($\zeta = 0$) if $r_s > (2\pi/5)(9\pi/4)^{1/3}(2^{1/3} + 1) = 5.45$.

(c) In the low-density limit of $r_s \to \infty$, Wigner[1] has shown that the electron gas becomes crystallized in a so-called Wigner solid. Find an expression for the total energy of a Wigner solid, and determine the range of r_s for which the Wigner solid can be the ground state through the comparison with the results of (a). Discuss the physical origin of the Wigner solid.

(d) Based on the results obtained above, discuss what the remaining issues are as raised in the articles by D. Ceperley [Nature **397**, 386] and D. P. Young *et al.* [Nature **397**, 412].

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[1]E.P. Wigner, Phys. Rev. **46**, 1002 (1934).